

Remarks on the W propagator at the resonance

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Abstract

We address the problem of properly defining the W^\pm , Z^0 propagator in the resonance region. Particular attention is paid to the longitudinal piece of this propagator. We also discuss the related renormalization procedures and the unitarity property.

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There has been recently a renewed interest regarding the correct form of the massive gauge-boson propagators to be used in the resonance region [1-9]. In particular, the accuracy attained in LEP experiments regarding the Z^0 mass, has raised the question whether the extracted value of the renormalized on-shell Z^0 mass [10] is gauge-dependent in higher orders of perturbation theory [1,4,5]. Thus, it was proposed [1,2,4,5] to return to the definition of the mass and width of the resonance in terms of the real and imaginary parts of the pole position of the amplitude. On this side, it has been shown [1,4,5] that a Laurent expansion around the complex pole, provides a systematic way to maintain the gauge-invariance of the amplitude at any order of perturbation theory.

The situation concerning the W^\pm , Z^0 bosons has been also discussed in references [3,7-9]. In some sense, the W^\pm case is less complicated as far as effects related to the γZ mixing are not present; however, the $q_\mu q_\nu$ piece in the W^\pm propagator can have important effects, in contrast to the Z^0 case which usually appears coupled to light fermions. For instance, since $m_t > M_W + m_b$ one can easily realize from references [8,9,11,12] that the correct form of the W^\pm propagator, and in particular of the $q_\mu q_\nu$ piece, is important in order to assess the size for the CP asymmetry arising from interference effects between two *top* quark decay diagrams, one containing a resonant W propagator and the other involving a CP violating phase.

In a recent paper [3] we have proposed that the correct form of the lowest order $W^\pm(Z^0)$ renormalized propagator, in the unitary gauge, to be used in the resonance region should be

$$\overline{\Delta}_{\mu\nu}(q) = \frac{i \{-g_{\mu\nu} + q_\mu q_\nu / (M_W^2 - iM_W\Gamma_W)\}}{q^2 - M_W^2 + iM_W\Gamma_W}, \quad (1)$$

where the mass M_W and width Γ_W of the W boson are related (see Eq.(8) below) to the real and imaginary parts of the pole position, which is a basic property of S -matrix [13]. Our argument was based [3] in the fulfillment of the (lowest order) Ward identity involving the electromagnetic vertex $WW\gamma$. As it was mentioned in ref.[3], this identity assures that the amplitude for processes such as $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions} + \gamma$ or $t \rightarrow bl^+\nu_l\gamma$ are gauge-invariant under electromagnetism only by using Eq.(1) above (see also Appendix B). Furthermore, we mentioned [3] that to deal with an arbitrary ξ -gauge, the replacement $M_W^2 \rightarrow M_W^2 - iM_W\Gamma_W$ should be done everywhere the W mass appears in the usual Feynman rules [14].

Using a different line of arguments, the authors in refs. [7] get a propagator similar to Eq.(1). However, in refs. [8,9] a different conclusion is obtained. As far as the derivation in [3] is concerned, the authors in ref. [9] pointed out that our argument is not consistent, because the electromagnetic Ward identity used in our paper seems to involve the propagators and the $WW\gamma$ vertex at different orders.

By using a general ξ -gauge we show in this paper that, when taken at lowest order, the renormalized W^\pm propagator obtained from the Dyson summation indeed reproduces the resonant form suggested in refs. [3,7,8]. This result is obtained from the renormalized propagator through its Laurent expansion around the pole position; the non-resonant terms arising in this expansion are shown to be explicitly of higher orders in the relevant coupling constant. We also address some comments on renormalization and unitarity.

Let us start by setting our conventions. In the general ξ -gauge, $\xi=1,0$ and ∞ correspond to the Landau, Feynman-'t Hooft and unitary gauges, respectively. The bare W boson propagator is given by [14]:

$$P_{\mu\nu}^{(0)}(q) = \frac{i \{-g_{\mu\nu} + (1 - \xi)q_\mu q_\nu / (q^2 - \xi M^2)\}}{q^2 - M^2}, \quad (2)$$

where M denotes the bare W mass.

The unrenormalized W self-energy can be decomposed into different forms as follows:

$$i\Pi_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \Pi_T^{WW}(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_L^{WW}(q^2) \quad (3a)$$

$$= -g_{\mu\nu} \Pi_T(q^2) + q_\mu q_\nu \Pi_L(q^2) \quad (3b)$$

$$= g_{\mu\nu} F_1(q^2) + (q^2 g_{\mu\nu} - q_\mu q_\nu) F_2(q^2) \quad (3c)$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \bar{\epsilon}_T(q^2) + \frac{q_\mu q_\nu}{q^2} \bar{\epsilon}_L(q^2) \quad (3d)$$

where the Eqs.(3a)-(3d) are the parametrizations used in refs.[15,7,8,9], respectively. Note that in ref.[9] only the imaginary parts of $\bar{\epsilon}_{T,L}$ have been considered. For simplicity, (unless specified) in the following we will not write the q^2 -dependence in the scalar self-energy functions.

By choosing the first parametrization —Eq. (3a)—, the infinite sum of the 1PI bubble graphs give rise to the following full unrenormalized propagator [15]:

$$P_{\mu\nu}(q) = \frac{i}{q^2 - M^2 - \Pi_T^{WW}} \left\{ -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \cdot \frac{q^2 - \xi(q^2 + \Pi_L^{WW} - \Pi_T^{WW})}{q^2 - \xi M^2 - \xi \Pi_L^{WW}} \cdot \left[\frac{1 - \frac{a}{(1-\xi)q^2 + \xi(\Pi_T^{WW} - \Pi_L^{WW})}}{1 - \frac{a}{q^2 - \xi M^2 - \xi \Pi_L^{WW}}} \right] \right\} \quad (4)$$

where $a \equiv \left(\frac{\xi q^2}{M^2}\right) \cdot (\Pi^{W\phi})^2 / (q^2 - \xi M^2 - \Pi^{\phi\phi})$, and $\Pi^{W\phi}$, $\Pi^{\phi\phi}$ are the corresponding self-energies for the $W - \phi$ and $\phi - \phi$ fields [15]. ϕ is the would-be goldstone associated to the W boson.

It can be easily shown that the contributions of the would-be goldstones (the term within squared brackets in Eq.(4)) can be written as follows:

$$1 - \frac{a'\xi(q^2 - M^2 - \Pi_T^{WW})}{q^2(1 - \xi) + \xi(\Pi_T^{WW} - \Pi_L^{WW})} \quad (5)$$

where we have defined $a' = a/[(1-a)(q^2 - \xi M^2 - \xi \Pi_L^{WW})]$. Thus, the would-be goldstones will not give rise to a pole in the W -boson propagator (see below).

If we neglect the terms proportional to a' which are $q_\mu q_\nu$ terms of $\mathcal{O}(g^4(q^2 - M^2 - \Pi_T^{WW})^0)$ —where g is the SU(2) gauge coupling constant—, Eq. (4) becomes:

$$P_{\mu\nu}(q) = \frac{i}{q^2 - M^2 - \Pi_T^{WW}} \left\{ -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \cdot \frac{q^2(1 - \xi) + \xi(\Pi_T^{WW} - \Pi_L^{WW})}{q^2 - \xi M^2 - \xi \Pi_L^{WW}} \right\}. \quad (6)$$

Observe that if we use Eq.(3b) into Eq.(6) we obtain the result given in Eq.(20) of ref.[7], namely:

$$P_{\mu\nu}^{[7]}(q) = \frac{i}{q^2 - M^2 + \Pi_T} \left\{ -g_{\mu\nu} + \frac{q_\mu q_\nu(1 - \xi - \xi \Pi_L)}{q^2 - \xi(M^2 - \Pi_T + q^2 \Pi_L)} \right\}.$$

Instead, if we choose to work in the unitary gauge, Eqs.(6) and (3c) give rise to Eq.(9) of ref. [8]:

$$P_{\mu\nu}^{[8]}(q) = i \frac{\left[-g_{\mu\nu} + q_\mu q_\nu \left(\frac{1 - F_2}{M^2 + F_1} \right) \right]}{q^2 - M^2 - F_1 - q^2 F_2},$$

while using Eqs.(6) and (3d) one obtains in the unitary gauge:

$$P_{\mu\nu}^{[9]}(q) = i \frac{\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \left(\frac{q^2 + \bar{\epsilon}_T + \bar{\epsilon}_L}{M^2 + \bar{\epsilon}_L} \right) \right]}{q^2 - M^2 + \bar{\epsilon}_T}.$$

which reproduces Eq.(3) of ref.[9] when one takes the imaginary parts of $\bar{\epsilon}_{T,L}$.

The derivation of the last three equations starting from (6) shows that , before renormalization, the results of refs. [7,8,9] are equivalent since they differ only by the parametrizations used for the self-energies, Eqs.(3).

Now, Eq.(6) can be rewritten into the following form:

$$P_{\mu\nu}(q) = i \left\{ \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}}{q^2 - M^2 - \Pi_T^{WW}} - \frac{q_\mu q_\nu}{q^2} \cdot \frac{\xi}{q^2 - \xi M^2 - \xi \Pi_L^{WW}} \right\} \quad (7)$$

which means that only the transverse part of the propagator will develop a S -matrix pole after renormalization.

Following references [1–5] we replace the bare mass in Eq.(6) in terms of the pole in the propagator:

$$\widetilde{M}^2 \equiv M_W^2 - iM_W\Gamma_W \quad (8)$$

$$= M^2 + \overline{\Pi}_T^{WW}. \quad (9)$$

(In the following $\overline{\Pi}_i^{WW}$ is used for $\Pi_i^{WW}(\widetilde{M}^2)$.)

Next, we can expand around the pole \widetilde{M}^2 the denominator in Eq.(6); we obtain:

$$\begin{aligned} q^2 - M^2 - \Pi_T^{WW}(q^2) &= q^2 - \widetilde{M}^2 - \Pi_T^{WW}(q^2) + \overline{\Pi}_T^{WW} \\ &= Z^{-1}(q^2 - \widetilde{M}^2) \{1 + \mathcal{O}(g^2(q^2 - \widetilde{M}^2))\}. \end{aligned} \quad (10)$$

where $Z^{-1} \equiv 1 - \overline{\Pi}_T^{WW}$.

If we also expand the coefficient of $q_\mu q_\nu$ —the term within curly brackets in Eq.(6)—, we get:

$$\frac{1}{q^2} \cdot \frac{q^2(1 - \xi) + \xi \Delta}{q^2 - \xi(M^2 + \Pi_L^{WW})} =$$

$$\begin{aligned}
&= \frac{1}{\widetilde{M}^2} \left\{ 1 - \frac{\widetilde{M}^2(1 - \xi \overline{\Pi}_T^{WW}) + \xi \overline{\Delta}}{\widetilde{M}^2(1 - \xi) + \xi \overline{\Delta}} \left(\frac{q^2 - \widetilde{M}^2}{\widetilde{M}^2} \right) + \dots \right\} \\
&\simeq \frac{1 - \xi'}{q^2 - \xi' \widetilde{M}^2} \tag{11}
\end{aligned}$$

where $\Delta \equiv \Pi_T^{WW} - \Pi_L^{WW}$ and

$$\begin{aligned}
\xi' &\equiv \frac{(1 - \overline{\Pi}_T^{WW})\xi}{1 + \xi \left(\frac{\overline{\Delta}}{\widetilde{M}^2} - \overline{\Pi}_T^{WW} \right)} \\
&= Z^{-1} \xi [1 + \mathcal{O}(g^2)]
\end{aligned}$$

(see ref.[16]).

Finally, if we replace Eqs.(10) and (11) into Eq.(6) we get:

$$P_{\mu\nu} = iZ \left\{ \frac{-g_{\mu\nu} + (1 - \xi') \frac{q_\mu q_\nu}{q^2 - \xi' \widetilde{M}^2}}{q^2 - \widetilde{M}^2} + \dots \right\} \tag{12}$$

where again, the ellipsis denote non-resonant $q_\mu q_\nu$ terms of $\mathcal{O}(g^2)$.

Let us make some remarks about our results:

(i) As it has been already discussed in refs. [1,4], the complex constants $\Pi_T^{WW}(\widetilde{M}^2)$ in Eq.(9) and Z in Eq.(12) play the role of the mass counterterm and W -field renormalization, respectively. In the on-shell scheme [10], the corresponding quantities are real constants. One can easily go from one scheme to the other by neglecting terms of $\mathcal{O}(g^4)$ (see Appendix A and ref.[4]).

Note that to reach the result in Eq.(12) one must renormalize the ξ gauge parameter (see ref.[16]). Observe also that in order to correctly drawn the

(lowest order) resonant propagator from the renormalized one, it is essential to extract the wavefunction renormalization constant Z .

(ii) The term inside the curly brackets in Eq.(12) contains the unique pole of the renormalized propagator. The remaining (non-resonant) terms are of $\mathcal{O}(g^2)$. Thus, *at lowest order* we recover the W^\pm propagator in the resonance region for an arbitrary ξ - gauge. In other words, the lowest order propagator at the resonance is obtained by replacing M^2 in Eq.(2) by the complex pole \widetilde{M}^2 as already stated in our previous paper [3].

(iii) If we formally expand $\Pi_T^{WW}(\widetilde{M}^2)$ around the mass M_W^2 —Eq.(8)— and then compare the real and imaginary parts of Eqs.(8,9), we obtain the following relations:

$$M_W^2 = M^2 + \text{Re}\Pi_T^{WW}(M_W^2) + \dots \quad (13)$$

$$-M_W\Gamma_W = \text{Im}\Pi_T^{WW}(M_W^2) + \dots \quad (14)$$

where the ellipsis denote terms of $\mathcal{O}(g^4)$ since Γ_W starts at $\mathcal{O}(g^2)$. The above expansion reproduces the on-shell renormalized mass, Eq.(13), and the unitarity relation (14) when taken at leading order; note however that unitarity is a relation exactly valid order by order. It should be noted also that M_W^2 above denotes the real part of the pole position and not the renormalized mass in the on-shell scheme.

(iv) Finally, the unitarity and mass renormalization relations of ref.[8] are obtained by using Eq.(3c) and the leading order expressions for Eqs.(13,14). On the other hand, since in ref.[9] only the imaginary parts of the W self-energy has been included in the Dyson summation, only Eq.(14) is reproduced as it can be explicitly shown from the expression $\bar{\epsilon}_T \sim i\Gamma_W^0$ given in ref.[9].

Summarizing, in this paper we have shown that the lowest order W^\pm propagator in the resonant region is given in an arbitrary ξ gauge by the term inside the curly brackets in Eq.(12), i.e.:

$$\overline{\Delta}_{\mu\nu}(q) = \frac{i \left\{ -g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi(M_W^2 - iM_W\Gamma_W)} \right\}}{q^2 - M_W^2 + iM_W\Gamma_W}. \quad (15)$$

We have shown that this propagator is the leading term in the expansion of the renormalized propagator around the pole \widetilde{M}^2 and that the non-resonant terms are explicitly of $\mathcal{O}(g^2)$. The unique pole of the propagator in Eq.(15) is located in its transverse part as it can be shown by projecting out this equation. Finally, the Ward identity used in ref.[3] is consistent since it involves only lowest order quantities.

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Appendix A

In this appendix we discuss the relationship between the wavefunction renormalization in the on-shell scheme [10] and the corresponding quantity when we fix the pole in the propagator in the S -matrix pole scheme [1,4]. As we argued above in the text, one can move from one to the other scheme by neglecting terms of $\mathcal{O}(g^4)$.

For definiteness we consider the complete propagator for an scalar particle.

$$\frac{1}{D} = \frac{1}{q^2 - m_0^2 - \Pi(q^2)} \quad (A1)$$

where m_0 denotes its bare mass and $\Pi(q^2)$ its 1PI self-energy.

The on-shell renormalization [10] is obtained by expanding $Re\Pi(q^2)$ around the renormalized mass m_R^2 :

$$D = q^2 - m_0^2 - Re\Pi(m_R^2) - (q^2 - m_R^2)Re\Pi'(m_R^2) + \cdots - iIm\Pi(q^2) \quad (A2)$$

$$\simeq Z^{-1}\{q^2 - m_R^2 - iZIm\Pi(q^2)\} \quad (A3)$$

where $Z^{-1} \equiv 1 - Re\Pi'(m_R^2)$ and $m_R^2 = m_0^2 + Re\Pi(m_R^2)$ correspond to the on-shell wavefunction and mass renormalization. Notice that both renormalized quantities, Z^{-1} and m_R , are real.

If we choose to expand also $Im\Pi(q^2)$ in Eq.(A2) around m_R we get:

$$\begin{aligned} D &= \overline{Z}^{-1} \left\{ q^2 - m_R^2 - i\overline{Z}Im\Pi(m_R^2) \right\} \\ &= \overline{Z}^{-1} \left\{ q^2 - m_R^2 + im_R\Gamma_R \right\} \end{aligned} \quad (A4)$$

where the second relation above follows from unitarity.

Observe that now the wavefunction renormalization $\overline{Z}^{-1} = 1 - \Pi'(m_R^2)$, becomes a complex quantity. This quantity can be rewritten as follows:

$$\overline{Z}^{-1} = Z^{-1} \left\{ 1 - i \text{Im} \Pi'(m_R^2) \right\} + \mathcal{O}(g^4), \quad (A5)$$

where g is the relevant coupling constant of the scalar particle to the particles involved in the 1PI graph. Thus, up to terms of $\mathcal{O}(g^4)$, Eq.(A5) furnish the relationship between Eqs.(A3) and (A4) (see also ref.[4]).

In contrast to the real renormalization constants in the on-shell scheme [10], the requirement of a defined S -matrix pole in the propagator as in Eq.(A4) naturally involves a complex-valued wavefunction renormalization.

Appendix B

In this appendix we use a simple model to illustrate how a resonant propagator with an energy-dependent width (for example the one of Ref. [9]) leads to violations of gauge invariance.

Let us consider the s -wave $\pi^+\eta$ scattering: $\pi^+(p)\eta(q) \rightarrow \pi^+(p')\eta(q')$. Near $\sqrt{s} \sim 1$ GeV, this process is dominated by the $a_0^-(980)$ meson. If we assume an energy-dependent width for the a_0 propagator, the corresponding scattering amplitude can be written as follows:

$$\mathcal{M}^0 = \frac{-ig^2}{s - m^2 + im\Gamma(s)} \quad (B1)$$

where $s = (p + q)^2 = (p' + q')^2$ is the squared of the center of mass energy and g the $a_0\pi^+\eta$ coupling constant.

Now let us consider the corresponding radiative process: $\pi^+(p)\eta(q) \rightarrow \pi^+(p')\eta(q')\gamma(\epsilon, k)$, where ϵ and k denote the four-polarization vector and

four-momentum of the photon. For simplicity we introduce the kinematical variables $s = (p + q)^2$, $s' = (p' + q')^2$, such that $s = s' + 2(p + q) \cdot k$.

The scattering amplitude receives contributions from three sources: the emission of the γ from the π^+ external lines and the emission from the a_0^- line. The explicit form of the amplitude is:

$$\mathcal{M} = ieg^2 \epsilon^\mu \left\{ \frac{p_\mu}{p \cdot k} \cdot \frac{1}{s' - m^2 + im\Gamma(s')} - \frac{p'_\mu}{p' \cdot k} \cdot \frac{1}{s - m^2 + im\Gamma(s)} - \frac{2(p + q)_\mu}{[s - m^2 + im\Gamma(s)][s' - m^2 + im\Gamma(s')]} \right\} \quad (B2)$$

where e denotes the π^+ electric charge.

As is well known, electromagnetic gauge-invariance requires that $\mathcal{M} = 0$ when $\epsilon \rightarrow k$. Thus, Eq. (B2) satisfies gauge-invariance only if $\Gamma(s) = \Gamma(s')$, *i.e.* the width in the propagator has to be a constant.

In the same way, it is very easy to check that the amplitude for the process $t \rightarrow b\tau^+\nu_\tau\gamma$ is gauge invariant under electromagnetism only if Eq. (1) is used for the propagator of the virtual W^+ .

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